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MOVABILITY AND STRONG WHITNEY-REVERSIBLE PROPERTIES

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In [24, Theorem 4 and Example 3], Nadler proved that the property of being locally connected is a sequential strong Whitney-reversible property and the property of being pathwise connected is not a strong Whitney-reversible property. In this paper, we show that the property of being (pointed) 1-movable is a sequential strong Whitney-reversible property. As a corollary, the property of being (pointed) movable is a sequential strong Whitney-reversible property for curves. This is an affirmative answer to [8, (5.2)]. Also, we show that the property of being (nearly) 1-movable is a Whitney property.

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sequential strong Whitney-reversible property
(pointed) 1-movable   nearly 1-movable
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1. Introduction

In this paper, all spaces are assumed to be metric spaces and map means continuous function. Let X be a *continuum*, i.e., a compact connected metric space. By a *subcontinuum* of X we mean a nonempty compact connected subset of X. By the *hyperspace* of X we mean

$$C(X) = \{A \mid A \text{ is a subcontinuum of } X\}$$

with *Hausdorff metric* d_H (e.g., see [22, (0, 1)]). In 1932, Whitney [29, p. 275] showed that there exists a map $\omega : C(X) \to [0, \omega(X)]$ such that

(1) $\omega(\{x\}) = 0$ for $x \in X$, and
(2) if $A \subset B$ and $A \neq B$, then $\omega(A) < \omega(B)$.

If $\omega : C(X) \to [0, \omega(X)]$ is a map satisfying conditions (1) and (2) above, ω is called a *Whitney map*. It is well known that $\omega^{-1}(t)$ is a continuum for each t. A topological

Property P is said to be a *Whitney property* [20] provided that whenever a continuum X has Property P, so does $\omega^{-1}(t)$ for each Whitney map ω for $C(X)$ and each $t < \omega(X)$. Many authors have studied various kinds of Whitney properties.

In [23], Nadler introduced the notions of "strong Whitney-reversible property" and "sequential strong Whitney-reversible property." A topological Property P is said to be a *strong Whitney-reversible property* provided that whenever X is a continuum such that $\omega^{-1}(t)$ has Property P for some Whitney map ω for $C(X)$ and each $0 < t < \omega(X)$, then X has Property P. A topological Property P is said to be a *sequential strong Whitney-reversible property* provided that if X is a continuum such that there is a Whitney map ω for $C(X)$ and a sequence $\{t_n\}_{n=1}^{\infty}$ of positive numbers such that $\lim t_n = 0$ and $\omega^{-1}(t_n)$ has Property P for each n, then X has Property P. A number of (sequential) strong Whitney-reversible properties have been studied (e.g., see [8, 17, 22, 23 and 24]).

In [24, Theorem 4 and Example 3], Nadler proved that the property of being locally connected is a sequential strong Whitney-reversible property and the property of being pathwise connected is not a strong Whitney-reversible property.

A compactum X lying in the Hilbert cube $Q = [-1, 1]^{\infty}$ is said to be *movable* [2, p. 151] provided that for every neighborhood U of X in Q, there is a neighborhood V of X in U such that for any neighborhood W of X in Q, there is a homotopy $h: V \times I \to U$ satisfying the following conditions:

$$h(z, 0) = z, \quad h(z, 1) \in W \text{ for each } z \in V. \tag{$*$}$$

A compactum X lying in Q is said to be *n-movable* [2, p. 171] provided that for every neighborhood U of X in Q, there is a neighborhood V of X in U such that for any neighborhood W of X in Q, any finite polyhedron $|K|$ with $\dim|K| \leq n$ and any map $f: |K| \to V$, there is a homotopy $h: |K| \times I \to U$ satisfying the following conditions:

$$h(z, 0) = f(z), \quad h(z, 1) \in W \text{ for each } z \in |K|. \tag{$**$}$$

Similarly, "pointed movable" and "pointed n-movable" are defined for the pointed compactum (X, x_0). It is known that "1-movable" does not imply "pointed 1-movable" [5], but it is still unknown whether movable continua must be pointed movable. Movable and pointed 1-movable continua are pointed movable. In shape theory, "pointed 1-movability" plays the role of pathwise connectivity. In [8, (1.5)], we proved that the property of being pointed 1-movable is a Whitney property. Also, the property of being (pointed) 2-movable is not a Whitney property [12, (1.1)].

In this paper, we show that the property of being (pointed) 1-movable is a sequential strong Whitney-reversible property. As a corollary, the property of being (pointed) movable is a sequential strong Whitney-reversible property for curves. This is an affirmative answer to [8, (5.2)].

Also, we show that the property of being (nearly) 1-movable is a Whitney property.

We refer readers to [16] and [22] for hyperspace theory and to [2] for shape theory.

2. The property of being (pointed) 1-movable is a sequential strong Whitney-reversible property

In this section, we prove the following theorem.

2.1. Theorem. *Let X be a continuum and let ω be any Whitney map for $C(X)$. Let $\{t_n\}_{n=1}^{\infty}$ be a decreasing sequence of positive numbers such that* $\lim t_n = t$. *If $\omega^{-1}(t_n)$ is (pointed) 1-movable, then $\omega^{-1}(t)$ is (pointed) 1-movable. In particular, the property of being (pointed) 1-movable is a sequential strong Whitney-reversible property.*

To prove Theorem 2.1, we need the following statement.

2.2 (L.E. Ward, Jr. [28, (3.4)]). *Let X be a subcontinuum of a continuum Y and let ω be any Whitney map for $C(X)$. Then there is a Whitney map $\tilde{\omega}$ for $C(Y)$ which is an extension of ω.*

The following is easily proved. We omit the proof.

2.3. *Let X be a continuum and let ω be any Whitney map for $C(X)$. Then for any $\varepsilon > 0$ there is a $\delta > 0$ such that if $A, B \in C(X)$, $|\omega(A) - \omega(B)| < \delta$ and $B \subset U(A, \delta)$, then $d_{\mathrm{H}}(A, B) < \varepsilon$, where $U(A, \delta) = \{x \in X \mid d(x, A) < \delta\}$.*

If X is a Peano continuum, then X has a convex metric d. Let $K_d : C(X) \times [0, \infty] \to C(X)$ be the homotopy defined by

$$K_d(A, t) = \{x \in X \mid d(x, A) \leq t\} \quad \text{(see [7, (1.2)])}.$$

By using this homotopy K_d, we can easily obtain the following fact.

2.4. *Let X be a Peano continuum and let ω be any Whitney map for $C(X)$. Then for each $0 \leq s \leq t \leq u \leq \omega(X)$, $\omega^{-1}([t, u])$ is a strong deformation retract of $\omega^{-1}([s, u])$.*

Proof of Theorem 2.1. Suppose that $\{t_n\}_{n=1}^{\infty}$ is a decreasing sequence of positive numbers such that $\lim t_n = t$ and each $\omega^{-1}(t_n)$ is pointed 1-movable. We may assume that X is a subset of the Hilbert cube Q. By 2.2, there is a Whitney map $\tilde{\omega}$ for $C(Q)$ which is an extension of ω. Note that $C(Q)$ is an AR [30, p. 190]. Moreover $C(Q)$ is homeomorphic to Q [4].

Let U be any compact ANR neighborhood of $\omega^{-1}(t)$ in $C(Q)$. Since U is an ANR, there is a positive number ε' such that if Z is a subset of U with diam $Z < 5\varepsilon'$, then Z is contractible in U (see [1, p. 87]). Also, choose a positive number ε ($\varepsilon < \varepsilon'$) such that if $f, g : Z \to U$ are any maps with $d(f, g) < 2\varepsilon$, then f is ε'-homotopic to g in U. Choose a natural number N such that $\omega^{-1}([t, t_{N-1}]) \subset \operatorname{Int} U$ and if $A, B \in C(X)$, $B \subset U(A, \delta)$ ($\delta = t_{N-1} - t$) and $|\tilde{\omega}(A) - \tilde{\omega}(B)| < \delta$, then $d_{\mathrm{H}}(A, B) < \varepsilon$ (see (2.3)). Let $A_0 \in \omega^{-1}(t)$. We shall show that $(\omega^{-1}(t), A_0)$ is pointed 1-movable. Choose a point $A_1 \in \omega^{-1}(t_N)$ such that $A_0 \subset A_1$. Since $(\omega^{-1}(t_N), A_1)$ is pointed 1-movable, there is an ANR neighborhood V_1 of $\omega^{-1}(t_N)$ in U satisfying the desired conditions.

Then we can choose a neighborhood V' of X in $Q=\tilde{\omega}^{-1}(0)$ such that V' is a Peano continuum and $\tilde{\omega}^{-1}(t_N)\cap C(V')\subset \operatorname{Int} V_1$. Set $V=\tilde{\omega}^{-1}([t-s, t_N])\cap C(V')$, where s is a sufficiently small positive number such that $V\subset U$. Let W be any neighborhood of $\omega^{-1}(t)$ in U. Choose a neighborhood W' of X in Q such that $\tilde{\omega}^{-1}(t)\cap C(W')\subset W$. We may assume that W' is a Peano continuum. Let f: $(|K|,*)\to(V, A_0)$ be any map, where K is a finite 1-dimensional simplicial complex and $*$ is a vertex of K. Since V' is a Peano continuum, there is a retraction r: $V\to\tilde{\omega}^{-1}(t_N)\cap C(V')$ such that $Z\subset r(Z)$ for each $Z\in V$ (see 2.4). Consider the map $r\cdot f:|K|\to\tilde{\omega}^{-1}(t_N)\cap C(V')\subset V_1$. Since $A_1\cap r\cdot f(*)\supset A_0$, there is a path $\alpha: I\to\tilde{\omega}^{-1}(t_N)\cap C(V')$ such that $\alpha(0)=r\cdot f(*)$, $\alpha(1)=A_1$ and $\operatorname{diam}\alpha(I)<2\varepsilon$ (see [24] and 2.3). By the homotopy extension theorem, we have a map $\varphi:(|K|,*)\to(V_1, A_1)$ such that φ is 2ε-homotopic to $r\cdot f$. Since $d(f, r\cdot f)<\varepsilon$, there is a $3\varepsilon'$-homotopy $h_1:|K|\times I\to U$ such that $h_1\||K|\times\{0\}=f$ and $h_1\||K|\times\{1\}=\varphi$.

Then we have a homotopy $h_2:(|K|,*)\times I\to(U, A_1)$ such that $h_2\||K|\times\{0\}=\varphi$ and $h_2(|K|\times\{1\})\subset\tilde{\omega}^{-1}([t_N-s'', t_N+s''])\cap C(W')$, where s'' is a sufficiently small positive number such that $t<t_N-s''$ and $t_N+s''\leq t_{N-1}$. Since W' is a Peano continuum, there is a positive number δ' ($\delta'<\delta$) such that if $a, b\in W'$ and $d(a,b)<\delta'$, there is a path $\beta: I\to W'$ from a to b such that $\operatorname{diam}\beta(I)<\delta$. Set $g=h_2\||K|\times\{1\}$. Choose a subdivision K' of K such that if $\langle v, w\rangle$ is a 1-simplex of K', then $\operatorname{diam} g(\langle v, w\rangle)<\delta'$. For each vertex v of K', choose a point $g'(v)$ of $\tilde{\omega}^{-1}(t)\cap C(W')$ such that $g'(v)\subset g(v)$. For each 1-simplex $\langle v, w\rangle$ of K', there is a path $\alpha: I\to W'$ from a point of $g(v)$ to a point of $g(w)$ such that $\operatorname{diam}\alpha(I)<\delta$. Consider the subcontinuum $g(v)\cup\alpha(I)\cup g(w)$ of W' and choose a sufficiently small neighborhood M of $g(v)\cup\alpha(I)\cup g(w)$ in W' such that M is a Peano continuum and $\operatorname{diam}[\tilde{\omega}^{-1}(t)\cap C(M)]<\varepsilon$ (see 2.3). Since $\tilde{\omega}^{-1}(t)\cap C(M)$ is a Peano continuum, there is a path g': $\langle v, w\rangle\to\tilde{\omega}^{-1}(t)\cap C(M)$ from $g'(v)$ to $g'(w)$. Hence, we have a map $g':|K|\to\tilde{\omega}^{-1}(t)\cap C(W')$ such that $d(g, g')<2\varepsilon$. We may assume that $g'(*)=A_0$. Hence we have an ε'-homotopy $h_3:|K|\times I\to U$ such that $h_3: g\simeq g'$. By using the homotopies h_1, h_2 and h_3, we obtain a homotopy $H:|K|\times I\to U$ such that $H: f\simeq g'$. Note that $\operatorname{diam} H(\{*\}\times I)<4\varepsilon'$, hence the loop $H|\{*\}\times(I,\dot{I})$ is contractible in U. By the homotopy extension theorem, we may assume that $H(\{*\}\times I)=A_0$. Also, $H(|K|\times\{1\})\subset\tilde{\omega}^{-1}(t)\cap C(W')\subset W$. This implies that $(\omega^{-1}(t), A_0)$ is pointed 1-movable. Similarly, the case of "1-movable" can be proved. This completes the proof. □

2.5. Corollary. *The property of being (pointed) movable is a sequential strong Whitney-reversible property for curves (=1-dimensional continua).*

3. The property of being (nearly) 1-movable is a Whitney property

In [8, (1.5)], we proved that the property of being pointed 1-movable is a Whitney property. To prove this result, we used the notion of "joinability", which is equivalent to that of pointed 1-movability and which was introduced by Krasinkiewicz and

Minc [19]. It is known that pointed 1-movability is different from 1-movability [5]. In this section, we prove that the property of being (nearly) 1-movable is a Whitney property. By using the same method as in this paper, we can directly prove that the property of being pointed 1-movable is a Whitney property, without using the notion of joinability.

A compactum X lying in Q is said to be *nearly* 1-*movable* [21] provided that for every neighborhood U of X in Q, there is a neighborhood V of X in U such that for any loop $f:\partial D\to V$ and for any neighborhood W of X in U, there is a finite collection of disjoint discs D_i in $D-\partial D$ and an extension $\tilde{f}: D-\bigcup \operatorname{Int} D_i\to U$ of f such that $\tilde{f}(\bigcup \partial D_i)\subset W$, where D is a disc. It is known that the nearly 1-movable continua coincide with the LC^1-divisors (see [6]).

3.1. Theorem. *The property of being* (*nearly*) 1-*movable is a Whitney property.*

Proof. Let X be a continuum lying in Q and let ω be any Whitney map for $C(X)$. Suppose that X is nearly 1-movable. Let $t\in(0,\omega(X))$. By 2.2, there is a Whitney map $\tilde{\omega}$ for $C(Q)$ which is an extension of ω. Let U be any compact ANR neighborhood of $\omega^{-1}(t)$ in $C(Q)$. Choose a neighborhood U' of X in $Q=\tilde{\omega}^{-1}(0)$ such that $C(U')\cap\tilde{\omega}^{-1}(t)\subset \operatorname{Int} U$ and U' is a Peano continuum. Since X is nearly 1-movable, there is a neighborhood V' of X in Q satisfying the desired conditions. We may assume that V' is a Peano continuum.

Since U is an ANR, there is an $\varepsilon>0$ such that if $g_1, g_2: Z\to U$ are maps with $d(g_1,g_2)<2\varepsilon$, then $g_1\simeq g_2$ in U. By 2.3, we can choose $\delta>0$ such that if $A, B\in C(X)$, $|\tilde{\omega}(A)-\tilde{\omega}(B)|<\delta$ and $B\subset U(A,\delta)$, then $d_{\mathrm{H}}(A,B)<\varepsilon$. Set $V=\tilde{\omega}^{-1}([t-s,t+s])\cap C(V')$, where s is a sufficiently small positive number such that $V\subset U$ and $s<\delta$.

Let $f:\partial D\to V$ be any loop and let W be any neighborhood of $\omega^{-1}(t)$ in U. Since $\tilde{\omega}^{-1}([t,t+s])\cap C(V')$ is a strong deformation retract of V, we may assume that $f(\partial D)\subset\tilde{\omega}^{-1}([t,t+s])\cap C(V')$ (see 2.4). Choose a neighborhood W' of X in Q such that $\tilde{\omega}^{-1}(t)\cap C(W')\subset W$. We may assume that W' is a Peano continuum. Since V' is a Peano continuum, there is a $\delta'>0$ such that if $a,b\in V'$ and $d(a,b)<\delta'$, then there is a path $\alpha:I\to V'$ such that $\alpha(0)=a$, $\alpha(1)=b$ and $\operatorname{diam}\alpha(I)<\delta$. Choose a finite sequence $a_0,a_1,\dots,a_n,a_{n+1}=a_0$ of points of ∂D such that $\operatorname{diam} f([a_i,a_{i+1}])<\delta'$, where $[a_i,a_{i+1}]$ denotes the smaller arc from a_i to a_{i+1} in ∂D. For each $i=0,1,2,\dots,n$, choose a path $\alpha_i: I\to V'$ from a point of $f(a_i)$ to a point of $f(a_{i+1})$ such that $\operatorname{diam}\alpha_i(I)<\delta$, and choose a Peano continuum M_i in V' such that $f(a_i)\cup\alpha(I)\cup f(a_{i+1})\subset M_i$ and $U(f(a_i),\delta)\supset M_i$, where $U(f(a_i),\delta)$ denotes the δ-neighborhood about the set $f(a_i)$. Also, for each $i=0,1,2,\dots,n$, choose a point $g(a_i)$ of $f(a_i)$ and a path $g_i:[a_i,a_{i+1}]\to M_i$ from $g(a_i)$ to $g(a_{i+1})$. By using g_i, we have a loop $g:\partial D\to V'$ such that $g([a_i,a_{i+1}])\subset M_i$. Since X is nearly 1-movable, there is an extension $\tilde{g}: D-\bigcup \operatorname{Int} D_j\to U'$ of g such that $\tilde{g}(\partial D_j)\subset W'$, where D_j is a disk in $D-\partial D$ such that $D_j\cap D_k=\emptyset$ $(j\neq k)$. Take a segment $\beta_i: I\to\tilde{\omega}^{-1}([0,t])\cap C(f(a_i))$ $(i=0,1,2,\dots,n)$ such that $\beta_i(0)=g(a_i)$ and $\beta_i(1)\in\tilde{\omega}^{-1}(t)\cap C(f(a_i))$ (see [16, (2.3)]).

Since $\tilde{\omega}^{-1}(t)\cap C(M_i)$ is a strong deformation retract of $\tilde{\omega}^{-1}([0,t])\cap C(M_i)$ (see 2.4), there is a map $h_i:[a_i,a_{i+1}]\times I\to\tilde{\omega}^{-1}([0,t])\cap C(M_i)$ such that $h_i|[a_i,a_{i+1}]\times\{0\}=g|[a_i,a_{i+1}]$, $h_i([a_i,a_{i+1}]\times\{1\})\subset\tilde{\omega}^{-1}(t)\cap C(M_i)$, $h_i|\{a_i\}\times I=\beta_i$ and $h_i|\{a_{i+1}\}\times I=\beta_{i+1}$. By using h_i, we obtain a homotopy $H_1:\partial D\times I\to\tilde{\omega}^{-1}([0,t])\cap C(V')$ such that $H_l|[a_i,a_{i+1}]\times I=h_i$. Then we have $d(H_1|\partial D\times\{1\},f)<2\varepsilon$, hence $H_1|\partial D\times\{1\}\simeq f$ in U. Since $\tilde{\omega}^{-1}(t)\cap C(W')$ is a strong deformation retract of $\tilde{\omega}^{-1}([0,t])\cap C(W')$, there is a homotopy $H_2:(\bigcup\partial D_j)\times I\to\tilde{\omega}^{-1}([0,t])\cap C(W')$ such that $H_2|(\bigcup\partial D_j)\times\{0\}=\tilde{g}|(\bigcup\partial D_j)$ and $H_2((\bigcup\partial D_j)\times\{1\})\subset\tilde{\omega}^{-1}(t)\cap C(W')\subset W$. Since $\tilde{\omega}^{-1}(t)\cap C(U')$ is a strong deformation retract of $\tilde{\omega}^{-1}([0,t])\cap C(U')$, we have an extension $\tilde{f}:D-\bigcup\operatorname{Int}D_j\to U$ of f such that $\tilde{f}(\bigcup\partial D_j)\subset W$. This implies that $\omega^{-1}(t)$ is nearly 1-movable. Similarly, the case of "1-movable" is proved. (See also Fig. 1.) □

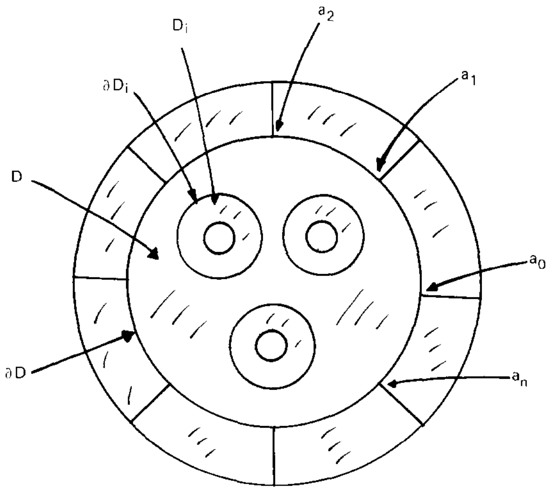

Fig. 1.

3.2. Theorem. *Let X be a continuum and let ω be any Whitney map for $C(X)$. Suppose that $\{t_n\}_{n=1}^{\infty}$ is a decreasing sequence of positive numbers such that* $\lim t_n=t$. *If $\omega^{-1}(t_n)$ is nearly 1-movable for each n, then $\omega^{-1}(t)$ is nearly 1-movable. In particular, the property of being nearly 1-movable is a sequential strong Whitney-reversible property.*

Proof. See the proof of Theorem 2.1. □

3.3. Remark. In the proofs of Theorems 2.1 and 3.1, the fact that each continuum is AC^0 (approximately 0-connected) is essential. (see [2, p. 145] for the definition of AC^n). In general, if X is a continuum with $\dim X\geq 2$, for any $\varepsilon>0$ there is a subcontinuum A of X such that A is not AC^1 and diam $A<\varepsilon$. Hence we cannot generalize the proofs of Theorems 2.1 and 3.1 to the case of n-movability ($n \geq 2$). In fact, it is known that Theorem 3.1 for the case of 2-movability is not true (see [12, (1.1)]).

3.4. Remark. It is known that the properties of being an FAR, acyclic, AC^n, having $\dim \leq n$ or $Fd \leq n$, etc., are sequential strong Whitney-reversible properties (see [8], [17] and [23]). Also, the properties of being an FANR or being calm are not strong Whitney-reversible properties (see [8]).

The following problem remains open.

3.5. Problem. Is the property of being (pointed) movable a (sequential) strong Whitney-reversible property? Let X be Borsuk's nonmovable Peano continuum [3]. Is there a Whitney map ω for $C(X)$ such that $\omega^{-1}(t)$ is movable (or 2-movable) for each $t > 0$?

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